Recall from class that functions are equal if and only if they are equal on all inputs (this equality is called extensionality.)

**Definition 0.1. (extensionality)** If \( f, g \in A \rightarrow B \),
\[
    f = g \overset{\text{def}}{=} \forall x: A. f(x) = g(x)
\]

So, we can prove two functions \( f \) and \( g \) are equal by choosing an arbitrary \( x \in A \) and showing \( f(x) = g(x) \).

For example, if \( f(x) = |x| \) (the absolute value) and \( g(x) = x \) then, \( f \neq g \) when we consider them as functions in the type \( \mathbb{Z} \rightarrow \mathbb{Z} \) since \( f(-2) = 2 \) and \( g(-2) = -2 \). But, if we think of these functions as elements of \( \mathbb{N} \rightarrow \mathbb{N} \), they are equal. To see this, choose an arbitrary \( x \in \mathbb{N} \) and argue that \( f(x) = g(x) \) i.e. that \( |x| = x \). But this is trivially true when \( x \geq 0 \), which follows because \( x \in \mathbb{N} \).

**Problem 0.1.** Create a separate Haskell script called Plus.hs which includes definitions for the following functions.

\[
\begin{align*}
    \text{plus} &:: (\text{Integer}, \text{Integer}) \rightarrow \text{Integer} \\
    \text{plus}(x, y) &= x + y
\end{align*}
\]

\[
\begin{align*}
    \text{plusc} &:: \text{Integer} \rightarrow (\text{Integer} \rightarrow \text{Integer}) \\
    \text{plusc } x \ y &= x + y
\end{align*}
\]

Use \( \text{plusc} \) to create a function of type \( (\text{Integer} \rightarrow \text{Integer}) \) that adds 7 to its argument.

\( \text{plusSeven} = ??? \)

Add this function to the Plus module and test it in the interpreter.

Now, consider the following two definitions.

\[
\begin{align*}
    \text{compose} &: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow (a \rightarrow c) \\
    \text{compose } f \ g \ x &= f (g \ x) \\
    \text{id} &:: a \rightarrow a \\
    \text{id} \ x &= x
\end{align*}
\]

**Problem 0.2.** Implement these functions in a module than includes the Plus module and, in the interpreter, evaluate the following:

\[
\begin{align*}
    &: t \ \text{compose} \ \text{plusSeven} \ \text{plusSeven} \\
    &\ (\text{compose} \ \text{plusSeven} \ \text{plusSeven})0 \\
    &\ (\text{compose} \ \text{plusSeven} \ \text{plusSeven})1 \\
    &\ (\text{compose} \ \text{plusSeven} \ \text{plusSeven})2
\end{align*}
\]

We will write \( f \circ g \) instead of \( \text{compose } f \ g \).

**Problem 0.3.** Prove the following theorem.

[compose-id-right] For every function \( f \), if \( f \in A \rightarrow B \) then \( f \circ \text{id} = f \).

**Problem 0.4.** Prove the following theorem using extensionality.

[compose-id-left] For every function \( f \), if \( f \in A \rightarrow B \) then \( \text{id} \circ f = f \).