Other Evolutionary Approaches

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Evolutionary Approaches

- Genetic Algorithm
- Evolutionary Strategy
- Genetic Programming
Genetic Programming
Introduction

Genetic programming evolves computer program, based on the fact that any computer program is a sequence of operations applied to values.
A Tree = An Algebraic Expression
Example: Pythagorean Theorem

\[ c = ? \]
1. Initialize a Population of Expressions for “c”
2. Define Fitness Function
Compare Calculated Value with Fact

\[ c = (a \times b - b) + \sqrt{\frac{a}{b}} \]

\[ c' = 10.2 \]
Measure Fitness over Multiple Facts

<table>
<thead>
<tr>
<th>Side $a$</th>
<th>Side $b$</th>
<th>Hypotenuse $c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>5.830952</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
<td>16.124515</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>18.110770</td>
</tr>
<tr>
<td>32</td>
<td>11</td>
<td>33.837849</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>5.000000</td>
</tr>
</tbody>
</table>

\[ c = (a \cdot b - b) + \sqrt{\frac{a}{b}} \]

\[
\begin{array}{c}
\sqrt{\frac{a}{b}} \\
\param{sqrt}
\end{array}
\begin{array}{c}
- \\
\param{-}
\end{array}
\begin{array}{c}
\cdot \\
\param{\cdot}
\end{array}
\begin{array}{c}
b \\
\param{b}
\end{array}
\begin{array}{c}
\div \\
\param{\div}
\end{array}
\begin{array}{c}
a \\
\param{a}
\end{array}
\begin{array}{c}
b \\
\param{b}
\end{array}
\]
3. Crossover with Probability
Q: Can we swap...
Q: Can we swap...
Q: Can we swap...
4. Mutation with Probability
Q: Can we mutate...
Q: Can we mutate...
Q: Can we mutate...
Q: Can we mutate...
Q: Can we mutate...
Summary
Simulation Result
Evolutionary Strategy
Revisit Genetic Algorithm

Find an integer \( x \in [0, 15] \) that maximize

\[
 f(x) = 15x - x^2
\]

Find two integers \( x, y \in [0, 15] \) that maximize

\[
 f(x, y) = (1 - x)^2 e^{-x^2 - (y+1)^2} - (x - x^3 - y^3) e^{-x^2 - y^2}
\]
Evolutionary Strategy

Idea: optimize multiple variables by repeatedly perturbing them to improve fitness.

\[
\text{max } f(x_1, x_2)
\]
(1+1) ES Algorithm

1. Generate a population of parent parameters of size $N$: $x_1, x_2, ..., x_N$

2. Calculate the solution associated with the parents: $X = f(x_1, x_2, ..., x_N)$

3. Create a population of offspring parameters:
   - $x'_1 = x_1 + \alpha (0, \delta)$
   - $x'_2 = x_2 + \alpha (0, \delta)$
   - $...$
   - $x'_N = x_N + \alpha (0, \delta)$

4. Calculate the solution associated with the offspring: $X' = f(x'_1, x'_2, ..., x'_N)$

5. Is $X'$ better than $X$?

   - Yes: Replace the parent population with the offspring population
   - No: Continue
(1+1) ES Algorithm

1. start a population of one parent.
(1+1) ES Algorithm

1. start a population of one parent.

2. evaluate fitness of parent.
(1+1) ES Algorithm

1. start a population of one parent.
2. evaluate fitness of parent.
3. perturb parent to a candidate offspring
(1+1) ES Algorithm

1. start a population of one parent.
2. evaluate fitness of parent.
3. perturb parent to a candidate offspring
4. evaluate fitness of candidate
(1+1) ES Algorithm

1. start a population of one parent.
2. evaluate fitness of parent.
3. perturb parent to a candidate offspring
4. evaluate fitness of candidate
5. if candidate is better, replace parent with it otherwise, go back to step 3
(1+1) ES Algorithm

1. start a population of one parent.
2. evaluate fitness of parent.
3. perturb parent to a candidate offspring
4. evaluate fitness of candidate
5. if candidate is better, update population  
   otherwise, go back to step 3
6. repeat 3-5 until convergence
[E] \[ \max f(x,y) = x^2 - y \]

1. initialize \((x,y) = (0,1)\)
2. evaluate \(f(x,y) = -1\)
3. perturb \((x,y) \rightarrow (0.5,0.8)\)
4. evaluate \(f(0.5,0.8) = -0.55\)
5. \(-0.55 > -1\) (better), update \((x,y) \leftarrow (0.5,0.8)\)
6. repeat 3-5 until convergence

Here we can directly use \(f(x,y)\) as the fitness function.
Q] \[ \text{max } f(x,y) = x^2 - y \]

1. initialize \((x,y) = (0,1)\)

2. evaluate \(f(x,y) = -1\)

3. perturb \((x,y) \rightarrow (0,2)\)

4. 

5. 

6. repeat 3-5 until convergence
[Q] max f(x,y) = x^2 - y

1. initialize (x,y) = (0,1)

2. evaluate f(x,y) = -1

3. perturb (x,y) → (0,2)

4. evaluate f(0,2) = -2

5. -2 < -1 (worse), no update, go back to step 3

6. repeat 3-5 until convergence
Performance Graph of (1+1)

Generate a population of parent parameters of size N:
\[ x_1, x_2, \ldots, x_N \]

Calculate the solution associated with the parents:
\[ X = f(x_1, x_2, \ldots, x_N) \]

Create a population of offspring parameters:
\[ x'_1 = x_1 + \alpha (0, \delta); \]
\[ x'_2 = x_2 + \alpha (0, \delta); \]
\[ \ldots \]
\[ x'_N = x_N + \alpha (0, \delta) \]

Calculate the solution associated with the offspring:
\[ X' = f(x'_1, x'_2, \ldots, x'_N) \]

Is \( X' \) better than \( X \)?

Replace the parent population with the offspring population
Add Gaussian noise to every variable.

\[ x'_i = x_i + \alpha(0, \delta) \]

\( \alpha(0, \delta) \) is normal distribution with mean 0 and variance \( \delta \).
[D] Multiple Parent
Multiple Parents with Recombination

Random initial population of $\mu$ individuals. (Attributes & Variances)

Recombination of Attributes & Variances
Creation of $\lambda$ individuals

Mutation of Attributes & Variances

Evaluation of $\lambda$'s (fitness function)

Selection of new $\mu$ individuals from ($\mu+\lambda$)

repeat N generations
Comparing GA and ES

Generate a population of parent parameters of size $N$:

$x_1, x_2, ..., x_N$

Calculate the solution associated with the parents:

$X = f(x_1, x_2, ..., x_N)$

Create a population of offspring parameters:

$x'_1 = x_1 + \alpha (0, \delta)$;

$x'_2 = x_2 + \alpha (0, \delta)$;

$\vdots$

$x'_K = x_K + \alpha (0, \delta)$

Calculate the solution associated with the offspring:

$X' = f(x'_1, x'_2, ..., x'_K)$

No

Is $X'$ better than $X$?