

Decide whether the following are true or false. Say why or why not, in doing so, rely on the definitions of \subseteq and $=$. Remember that \emptyset is the same as $\{\}$, we use it because in some contexts it is more readable.

Recall the formal definitions:

$$A \subseteq B \stackrel{\text{def}}{=} \forall x.(x \in A \Rightarrow x \in B)$$

$$A = B \stackrel{\text{def}}{=} \forall x.(x \in A \Leftrightarrow x \in B)$$

The informal interpretation of sets as kinds of sacks¹ in which you can put things may help with these problems. Under the sack interpretation, the empty set is a sack containing nothing. The set containing the empty set $\{\{\}\}$ (or $\{\emptyset\}$) is a sack that contains an empty sack. *Membership* ($x \in A$): means you can reach into the sack A and pull out an x – it does not mean that you can reach in and pull out a sack containing an x , but that you actually have to be able to grab an x from the sack. *Subset* ($A \subseteq B$), not to be confused with membership, means that everything that is in the sack A is also in the sack B , *e.g.* if it is possible to pull an x out of A then you can pull an x out of sack B . *Equality*: sacks A and B are equal if everything that is in A is also in B and everything that is in B is also in A .

You might like to read what Hein has to say on pages 13-15 of chapter one which is linked on the web-page. Ignore his claim on page 14 that $\{H, E, L, L, O\}$ is not a set – otherwise you might find it useful.

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|---|---|
| 1.) $1 \in \{1\}$ | 9.) $1 \subseteq \{1\}$ |
| 2.) $\{1\} \subseteq \{1\}$ | 10.) $\emptyset \in \{1\}$ |
| 3.) $\emptyset \subseteq \{1\}$ | 11.) $\emptyset \in \emptyset$ |
| 4.) $\emptyset \subseteq \emptyset$ | 12.) $\{1, 2\} \subseteq \{\emptyset, 1, 2\}$ |
| 5.) $\emptyset \in \{\emptyset, 1, 2\}$ | 13.) $\emptyset \in \{1, 2\}$ |
| 6.) $\emptyset \in \emptyset$ | 14.) $\{\emptyset\} = \emptyset$ |
| 7.) $\{\{\emptyset\}\} = \emptyset$ | 15.) $\{1, 1\} \in \{\{1\}\}$ |
| 8.) $\{\{1, 1\}\} \subseteq \{\{1\}\}$ | 16.) $\{1\} \subseteq \{\{1\}\}$ |

¹The sack interpretation means the sacks have the magical property that, if you put the same thing into the sack more than once and then you take one of them out, they're all gone.