HW 15 **Prof. Caldwell**
Due: 27 October 2009 **COSC 2300**

Recall the powerset axiom.

$$x \in \rho(S) \stackrel{\text{def}}{=} x \subseteq S$$

Exercise 0.1. Prove the following theorem, that the powerset relation is monotone.

Theorem 0.1. For all sets A and B, $A \subseteq B \Rightarrow \rho(A) \subseteq \rho(B)$.

You may use the following derived rule to simplify your proof if you choose to do a sequent proof. (Note that the $\mathcal{A}$ and $\mathcal{B}$ denote arbitrary sets.

$$\frac{\Gamma_1, \mathcal{A} \subseteq \mathcal{B}, \Gamma_2, x \in \mathcal{A}, \Gamma_3, x \in \mathcal{B} \;\vdash\; \Delta}{\Gamma_1, \mathcal{A} \subseteq \mathcal{B}, \Gamma_2, x \in \mathcal{A}, \Gamma_3 \;\vdash\; \Delta} \text{ subset-member}$$

As an alternative to a sequent proof you may also give a suitable detailed proof in English. In this case the derived subset-member rule can be translated by saying:

Since we know $\mathcal{A} \subseteq \mathcal{B}$ and we know $x \in \mathcal{A}$ we know $x \in \mathcal{B}$.

Exercise 0.2. Read chapter 6 on relations in of the class notes from pp. 91 - 104.

Definition 0.1 (less than)

$$lt = \{\langle x, y\rangle \in \mathbb{Z} \times \mathbb{Z} | \exists w. w \in \mathbb{N} \wedge w \neq 0 \wedge x + w = y\}$$

Definition 0.2 (greater than or equal)

$$ge = \{\langle x, y\rangle \in \mathbb{Z} \times \mathbb{Z} | \exists w. w \in \mathbb{N} \wedge x = y + w\}$$

Exercise 0.3. Prove the following.

i.) $\langle 5, 7\rangle \in lt$

ii.) $\langle 5, 7\rangle \notin ge$.

iii.) $\langle 5, 5\rangle \in ge$.