HW 18 **Prof. Caldwell**
Due: November 5 2009 **COSC 2300**

1 Equivalence Relations and equivalence classes

Read the notes regarding equivalence relations and properties of equivalence relations.

Definition 1.1 (equivalence relation) A relation $R \subseteq A \times A$ is an equivlence relation iff it is

reflexive	$\forall x{:}A.\ xRx$
symmetric	$\forall x, y{:}A.\ xRy \Rightarrow yRx$
transitive	$\forall x, y, z{:}A.\ (xRy \wedge yRz) \Rightarrow xRz$

Definition 1.2 (equivlence class) If $R \subseteq A \times A$ is an equivalence relation, the *R-equivalence class of x* is

$$[x]_R = \{y : A \mid xRy\}$$

Exercise 1.1. Prove that if $R \subseteq A \times A$ is an equivalence relation,

$$\forall x, y : A.\ \ xRy \Rightarrow [x]_R = [y]_R$$

Hint: You will need to use the fact that R is symmetric and transitive and remember that $[x]_R$ and $[y]_R$ are sets (How do you prove sets are equal?)

2 An Equivalence Relation on Fractions

Definition 2.1 ($\mathbb{N}^+$)

$$\mathbb{N}^+ = \mathbb{N} - \{0\} = \{1, 2, 3, \cdots\}$$

Definition 2.2 (fractions) The set of *fractions* ($\mathcal{F}$) is defined as:

$$\mathcal{F} \stackrel{\text{def}}{=} \mathbb{Z} \times \mathbb{N}^+$$

We will write fractions as $\frac{a}{b}$ instead of $\langle a, b \rangle$.

Let $\equiv_{\mathbb{Q}} \subseteq \mathcal{F} \times \mathcal{F}$ be defined as follows:

$$\frac{a}{b} \equiv_{\mathbb{Q}} \frac{c}{d} \stackrel{\text{def}}{=} ad = bc$$

where ad means a times d using ordinary multiplication.

Exercise 2.1. Using ordinary properties of multiplication and division to prove that $\equiv_{\mathbb{Q}}$ is an equivalence relation. You'll have to show the following

reflexive: $\forall \frac{a}{b} : \mathcal{F}.\ \frac{a}{b} \equiv_{\mathbb{Q}} \frac{a}{b}$

symmetric: $\forall \frac{a}{b}, \frac{c}{d} : \mathcal{F}.\ \frac{a}{b} \equiv_{\mathbb{Q}} \frac{c}{d} \Rightarrow \frac{c}{d} \equiv_{\mathbb{Q}} \frac{a}{b}$

transitive: $\forall \frac{a}{b}, \frac{c}{d}, \frac{e}{f} : \mathcal{F}.\ (\frac{a}{b} \equiv_{\mathbb{Q}} \frac{c}{d} \wedge \frac{c}{d} \equiv_{\mathbb{Q}} \frac{e}{f}) \Rightarrow \frac{a}{b} \equiv_{\mathbb{Q}} \frac{e}{f}$