HW 2 **Prof. Caldwell**
Due: 3 September 2009 **COSC 2300**

Read pages 22 – 28 from the lecture notes.
Recall the definition of the valuation function.

$val(\alpha, \bot)$	$= \mathbf{F}$	
$val(\alpha, x)$	$= \alpha(x)$	whenever $x \in \mathcal{V}$
$val(\alpha, \neg\phi)$	$= \mathbf{not}(val(\alpha, \phi))$	
$val(\alpha, \phi \wedge \psi)$	$= \mathbf{and}(val(\alpha, \phi),\ val(\alpha, \psi))$	
$val(\alpha, \phi \vee \psi)$	$= \mathbf{or}(val(\alpha, \phi),\ val(\alpha, \psi))$	
$val(\alpha, \phi \Rightarrow \psi)$	$= \mathbf{not}\ val(\alpha, \phi)\ \mathbf{or}\ val(\alpha, \psi)$	

Also, the connectives have the following rules for evaluation.

$$\begin{array}{lcl} \mathbf{not}(\mathbf{T}) & = & \mathbf{F} \\ \mathbf{not}(\mathbf{F}) & = & \mathbf{T} \end{array} \qquad \begin{array}{lcl} \mathbf{and}(\mathbf{T}, \mathbf{T}) & = & \mathbf{T} \\ \mathbf{and}(\mathbf{F}, \mathbf{T}) & = & \mathbf{F} \\ \mathbf{and}(\mathbf{T}, \mathbf{F}) & = & \mathbf{F} \\ \mathbf{and}(\mathbf{F}, \mathbf{F}) & = & \mathbf{F} \end{array} \qquad \begin{array}{lcl} \mathbf{or}(\mathbf{T}, \mathbf{T}) & = & \mathbf{T} \\ \mathbf{or}(\mathbf{F}, \mathbf{T}) & = & \mathbf{T} \\ \mathbf{or}(\mathbf{T}, \mathbf{F}) & = & \mathbf{T} \\ \mathbf{or}(\mathbf{F}, \mathbf{F}) & = & \mathbf{F} \end{array}$$

$$\begin{array}{lcl} \mathbf{if}\ \mathbf{T}\ \mathbf{then}\ b_1\ \mathbf{else}\ b_2 & = & b_1 \\ \mathbf{if}\ \mathbf{F}\ \mathbf{then}\ b_1\ \mathbf{else}\ b_2 & = & b_2 \end{array}$$

1.)Compute the following valuations.

a. $val(\alpha, p \Rightarrow q)$ where

$\alpha(p) = \mathbf{T}$
$\alpha(q) = \mathbf{F}$

b. $val(\alpha, ((p \wedge q) \Rightarrow r) \Rightarrow ((p \Rightarrow r) \vee (q \Rightarrow r)))$ where

$\alpha(p) = \mathbf{T}$
$\alpha(q) = \mathbf{F}$
$\alpha(r) = \mathbf{F}$

2.) Extend the definition of *val* to include the formula $\phi \Leftrightarrow \psi$ where this new connective (called "if and only if") is defined as follows

$$\phi \Leftrightarrow \psi \quad \stackrel{\text{def}}{=} \quad ((\phi \Rightarrow \psi) \wedge (\psi \Rightarrow \phi))$$