HW 4 **Prof. Caldwell**
Due: 10 September 2009 **COSC 2300**

Recall the following definitions.

Definition 0.1 (iterative conjunction and disjunction of formula lists)

$$\bigwedge_{\phi\in[\,]} \phi = \neg\bot$$
$$\bigwedge_{\phi\in(h::t)} \phi = h \wedge (\bigwedge_{\phi\in t} \phi)$$

$$\bigvee_{\phi\in[\,]} \phi = \bot$$
$$\bigvee_{\phi\in(h::t)} \phi = h \vee (\bigvee_{\phi\in t} \phi)$$

Definition 0.2 (sequent) A *sequent* is a pair of lists of formula, $\langle \Gamma, \Delta \rangle$. We write the sequent $\langle \Gamma, \Delta \rangle$ as $\Gamma \vdash \Delta$. The formula list Γ is called the *antecedent* of the sequent and the formula list Δ is called the *succedent*. Read the sequent $(\vdash \Delta)$ as the pair $\langle [\,], \Delta \rangle$ and the sequent $(\Gamma \vdash)$ as the pair $\langle \Gamma, [\,] \rangle$

Definition 0.3 (meaning of a sequent) The *meaning of a sequent* ($[\![\Gamma \vdash \Delta]\!]$) is a formula defined as follows.

$$[\![\Gamma \vdash \Delta]\!] = (\bigwedge_{\phi\in\Gamma} \phi) \Rightarrow (\bigvee_{\psi\in\Delta} \psi)$$

Definition 0.4 (validity of a sequent) A sequent S is *valid* iff $\models [\![S]\!]$ (*i.e.* if the meaning of the sequent is a valid formula).

Assignment:

Calculate the meanings of the following sequents and determine if they are valid or not by using a truth table.

1.) $p \wedge q \vdash p$

2.) $p, q \vdash q \wedge p$

3.) $p, \neg p \vdash$ *i.e.* the sequent $\langle [p, \neg p], [\,] \rangle$.