

1 Derived Rules

In class we derived rules $\neg L$, $\neg R$ for negation on the left and negation on the right using a definition of negation given as follows:

$$\neg\phi \stackrel{\text{def}}{=} \phi \Rightarrow \perp$$

We also derived rules $\phi \Leftrightarrow \psi \Leftrightarrow L$ for if-and-only-if (iff) in the left and on the right using the definition:

$$\phi \Leftrightarrow \psi \stackrel{\text{def}}{=} (\phi \Rightarrow \psi) \wedge (\psi \Rightarrow \phi)$$

Problem 1.1. Derive proof rules for the *nand* (\otimes) and *nor* (\oplus) operators whose definitions are given as follows:

$$\begin{aligned}\phi \otimes \psi &\stackrel{\text{def}}{=} \neg(\phi \wedge \psi) \\ \phi \oplus \psi &\stackrel{\text{def}}{=} \neg(\phi \vee \psi)\end{aligned}$$

Problem 1.2. Use your proof rules to prove the following sequent:

$$\vdash \neg(p \otimes q) \Leftrightarrow \neg p \oplus \neg q$$