HW 13 **Prof. Caldwell**

Due: 7 October 2010 **COSC 3015**

Recall the list induction principle to prove a proeprty for finite lists of type a.

$$\begin{array}{l} [P([\,])\wedge \\ \ \forall x :: a. \forall xs :: [a].\, P(xs) \Rightarrow P(x : xs) \\ \ \ \Rightarrow \forall ys :: [a].P(ys) \end{array}$$

Thus, for a property P of lists, to show that $\forall ys :: [a].\, P(ys)$ it is enough to show two things:

$$\begin{array}{rl} \textbf{i.)} & P([\,]) \\ \textbf{ii.)} & \forall x :: a. \forall xs :: [a].\, P(xs) \Rightarrow P(x : xs) \end{array}$$

Here are some definitions.

$$\begin{array}{l} head(h : t) = h \\ head[\,] = \bot \\ \\ last[x] = x \\ last(h : t) = last\ t \\ last[\,] = \bot \\ \\ reverse[\,] = [\,] \\ reverse(h : t) = (reverse\ t) + +[h] \\ \\ map\ f\ [\,] = [\,] \\ map\ f\ (h : t) = (f\ h)\ :\ map\ f\ t \\ \\ (f\ .\ g)\ x = f\ (g\ x) \end{array}$$

Prove the following by list induction.

$$\begin{array}{rl} 1.) & \forall m\!:\![a].\ map\,(f\ .\ g)\,m = ((map\ f)\,.\,(map\ g))\ m \\ 2.) & \forall m\!:\![a].\ head\,(reverse\ m) = last\ m \\ 3.) & \forall m\!:\![a].\ last\,(reverse\ m) = head\ m \end{array}$$