HW 14 **Prof. Caldwell**
Due: 30 October 2007 **COSC 3015**

Recall the list induction principle.

$$\begin{array}{l}[P([\,])\wedge \\ \ \forall a:\alpha.\forall m:[\alpha].\,P(m)\Rightarrow P(a:m)] \\ \ \ \Rightarrow \forall m:[\alpha].P(m)\end{array}$$

Thus, for a property P of lists, to show that $\forall m:[\alpha].\,P(m)$ it is enough to show two things:

i.) $P([\,])$
ii.) $\forall a:\alpha.\forall m:[\alpha].\,P(m)\Rightarrow P(a:m)$

Here are some definitions.

$$\begin{array}{l} head(h:t) = h \\ head[\,] = \bot \\ \\ last[x] = x \\ last(h:t) = last\ t \\ last[\,] = \bot \\ \\ reverse[\,] = [\,] \\ reverse(h:t) = (reverse\ t) ++[h] \\ \\ map\ f\ [\,] = [\,] \\ map\ f\ (h:t) = (f\ h)\ :\ map\ f\ t \\ \\ (f\,.\,g)\ x = f\ (g\ x)\end{array}$$

Prove the following by list induction.

1.) $\forall m\!:\![\alpha].\ map\,(\backslash x\rightarrow x)\ m = m$
2.) $\forall m\!:\![\alpha].\ map\,(f\,.\,g)\,m = ((map\ f)\,.\,(map\ g))\ m$
3.) $\forall m\!:\![\alpha].\ head\,(reverse\ m) = last\ m$
4.) $\forall m\!:\![\alpha].\ last\,(reverse\ m) = head\ m$