Building Lemmas Using Examples

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Grupo de Procesamiento de Lenguaje Natural
FaMAF - UNC

International Workshop ACL2 - Nov. 2007

Overview

General Idea

1. Generates lemmas for helping proving theorems that state the equality between terms.

```
(defthm th1 (equal (f x) (g x)))
```

2. Selects a ground term a
3. Observes the computations of `(f a)` and `(g a)`
4. Suggests lemmas based on the observation.

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Finding Ground Terms: How do we find interesting examples?

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Finding Ground Terms: How do we find interesting examples? So far, by hand.

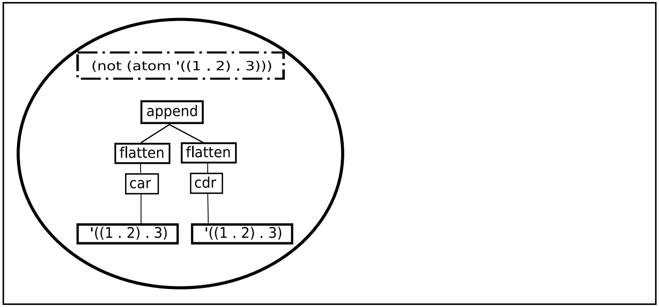
Computational Model: Where to observe computations? We introduce the concept of Evaluation Graphs.

Meaningful Observations: What are we looking for on this models? We look for a specific pattern while comparing Evaluation Graphs.

Basic Intuitions on EvalGraphs

EvalGraphs are directed graphs where

Vertices are pairs of terms. One of them is boolean.



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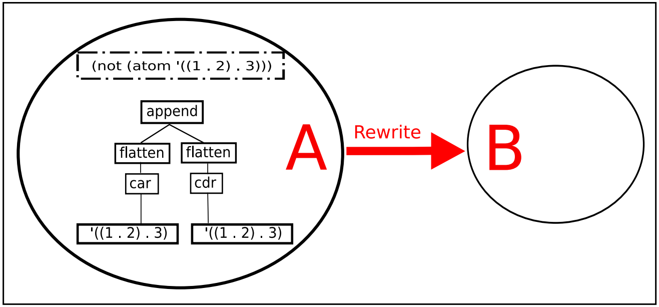
Labeled Arcs	(A, B) is an arc, if vertex B is the result of *expanding* or *rewritting* A



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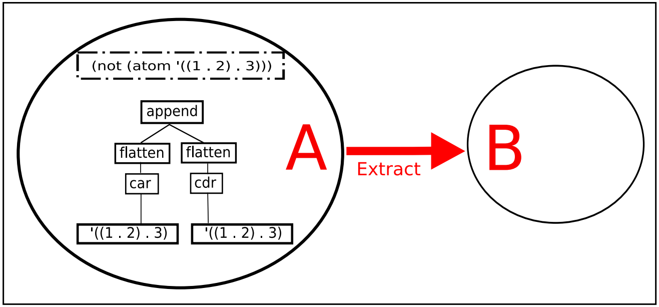

Operations on Terms: Rewrite

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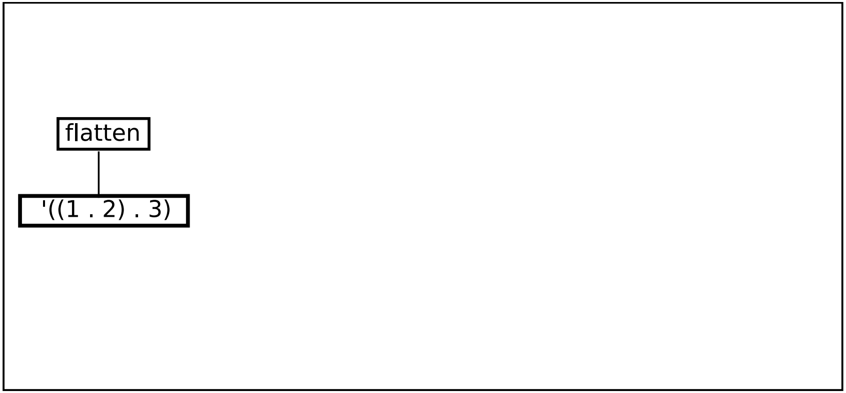

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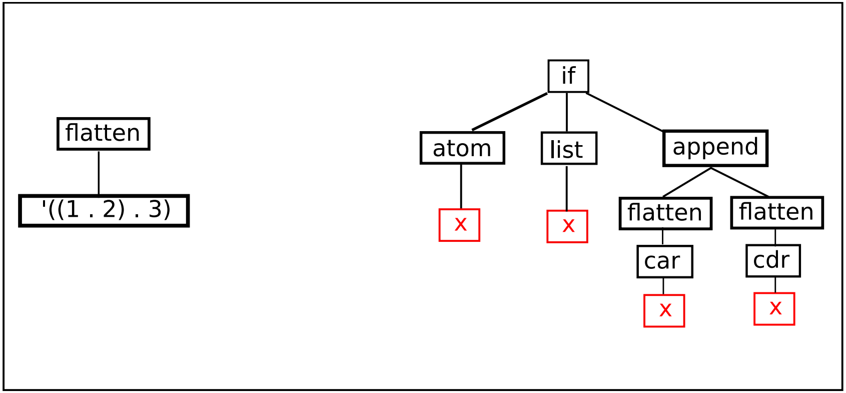

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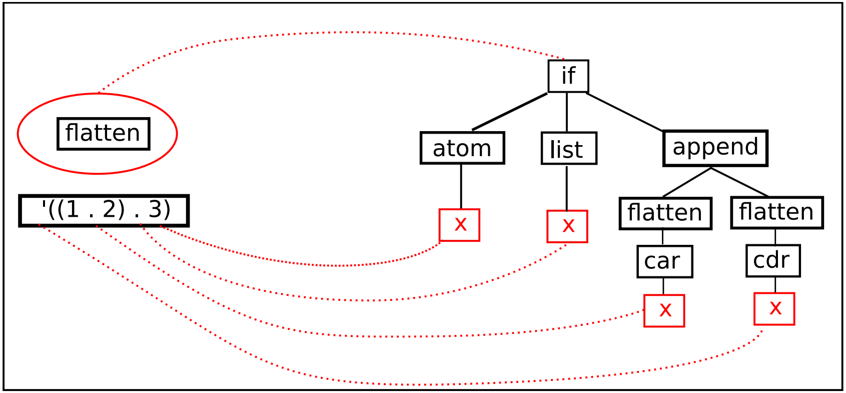

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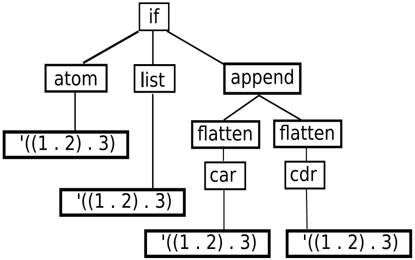

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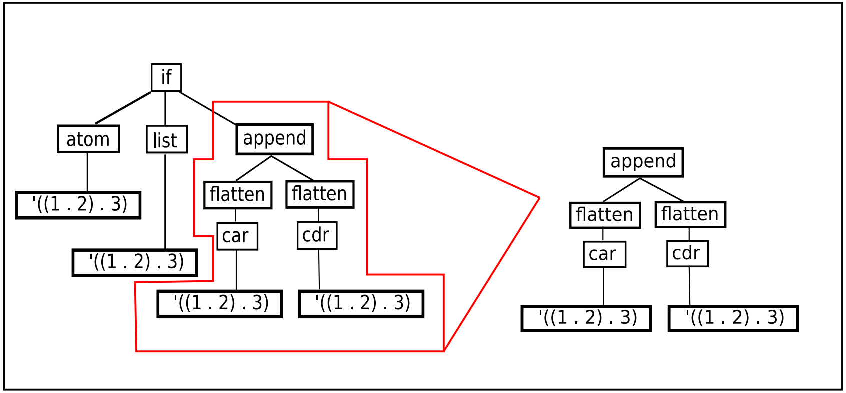

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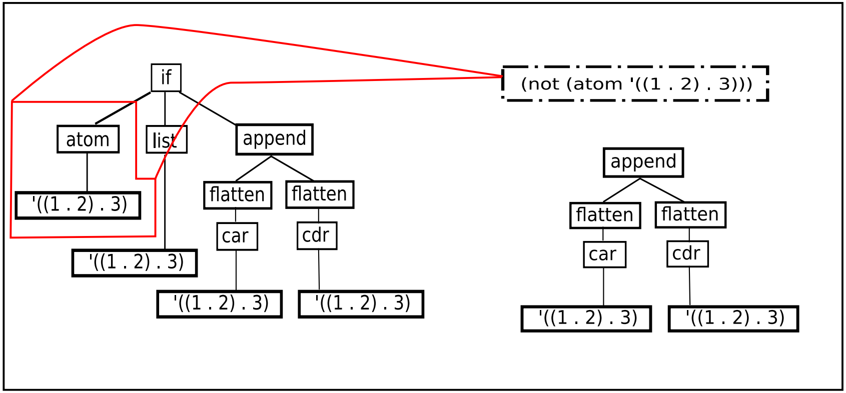

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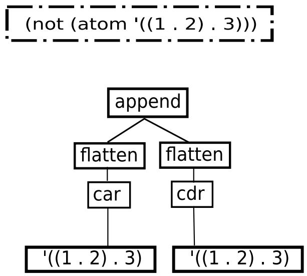

Operations on Terms: Extract

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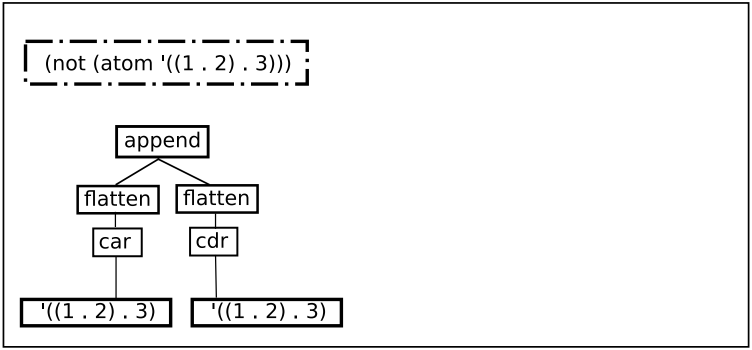

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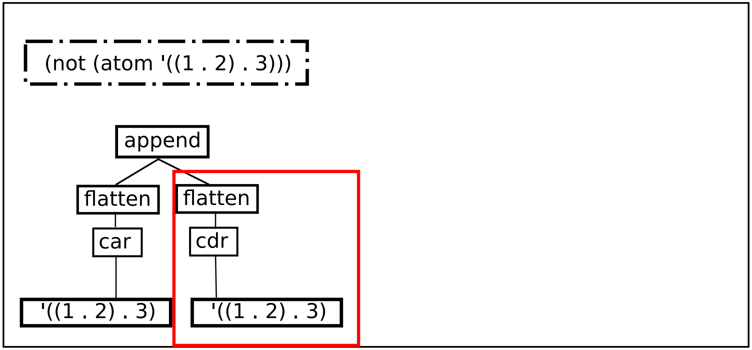

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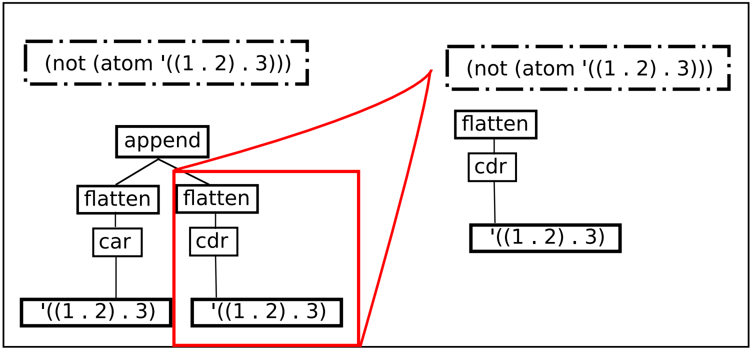
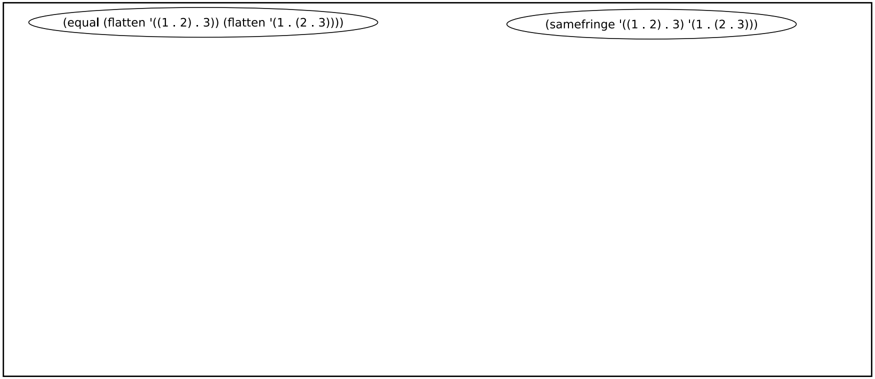


Meaningful Observations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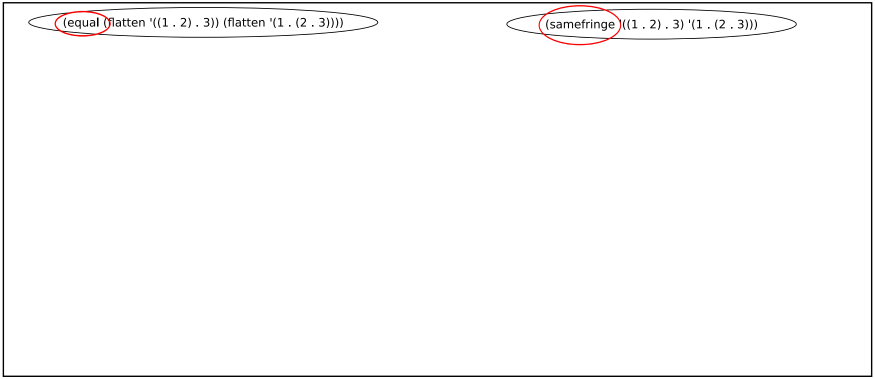
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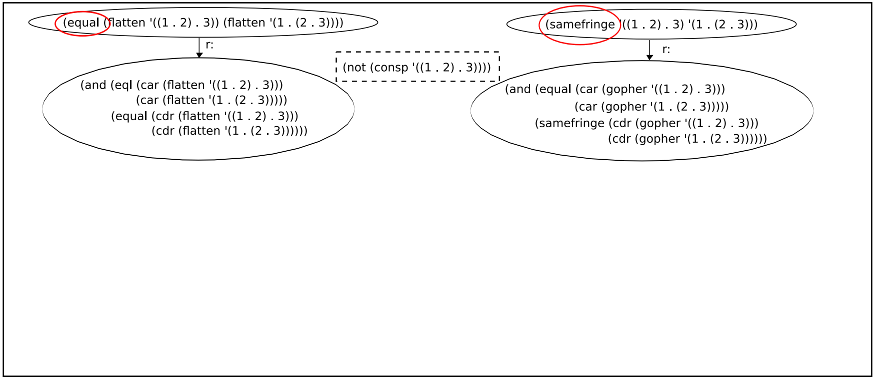
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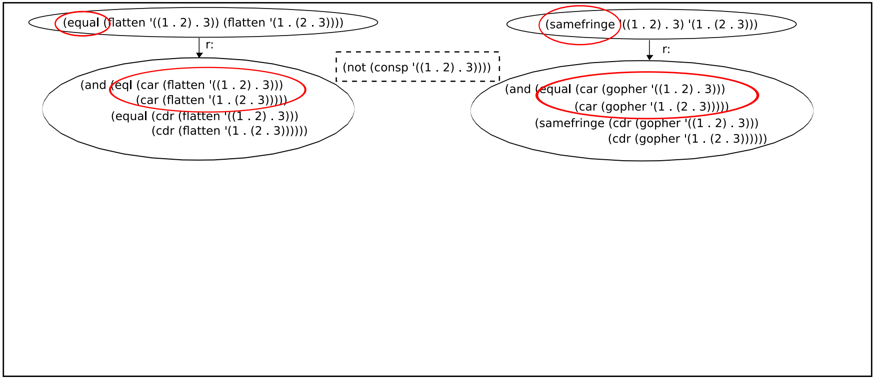

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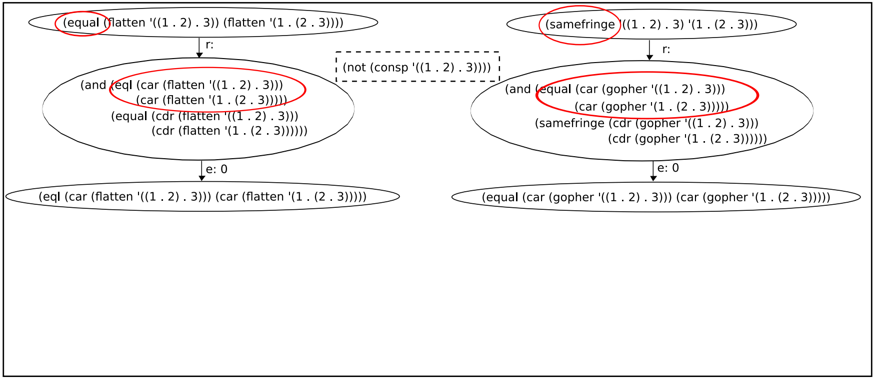
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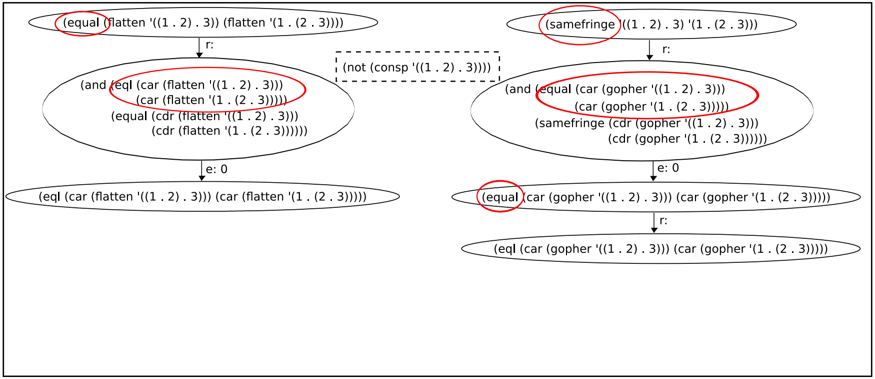
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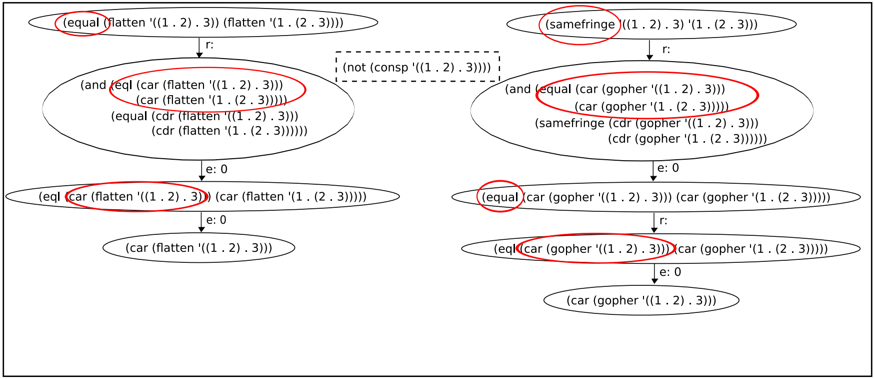
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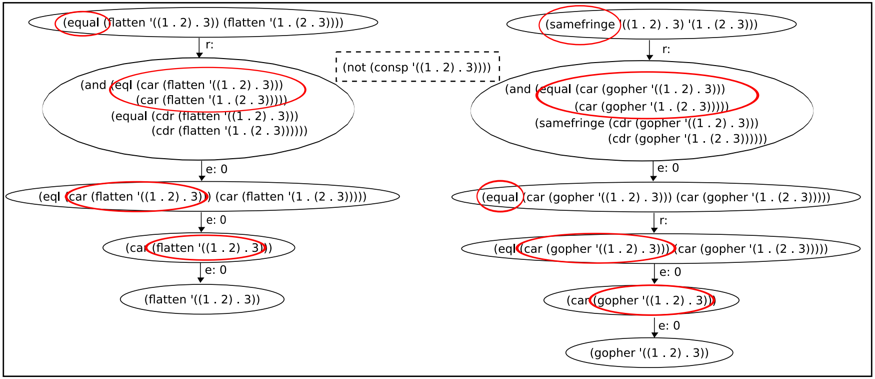
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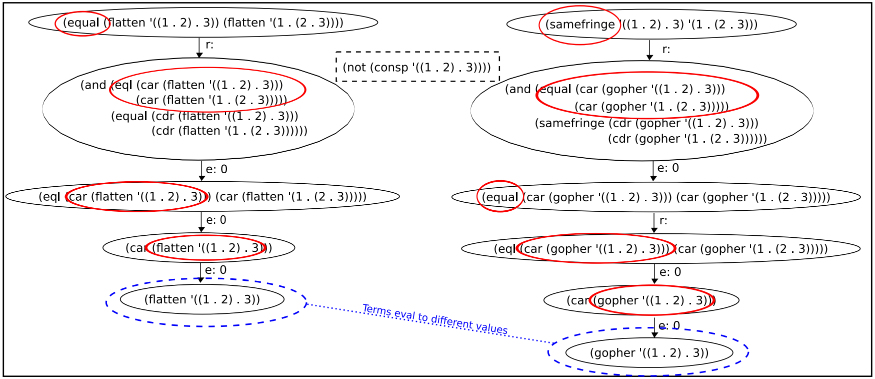
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Terms eval to different values
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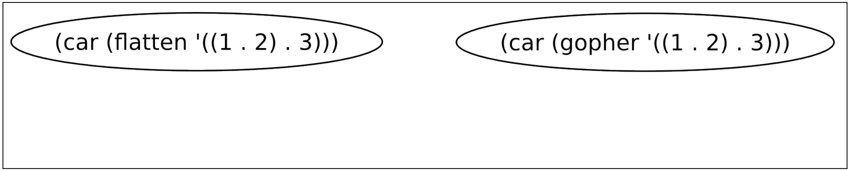

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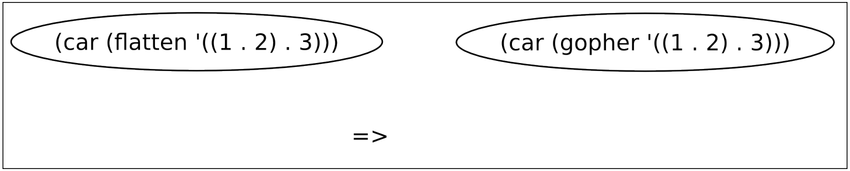

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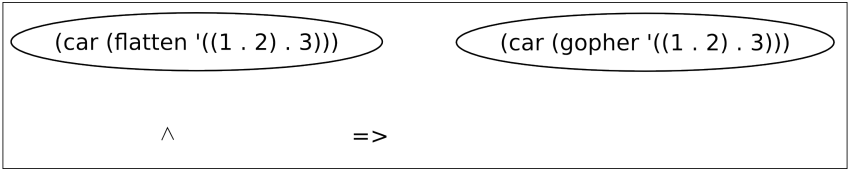

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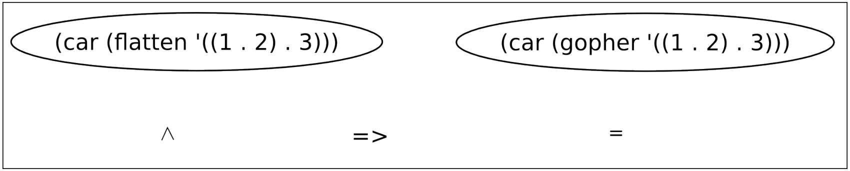

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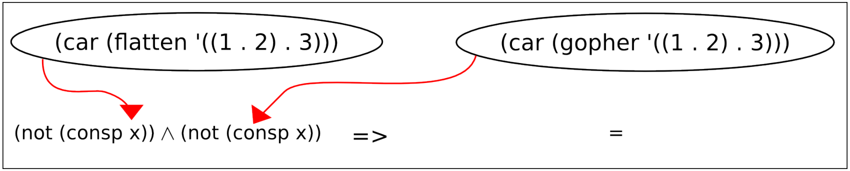
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(not (consp x)) ∧ (not (consp x))
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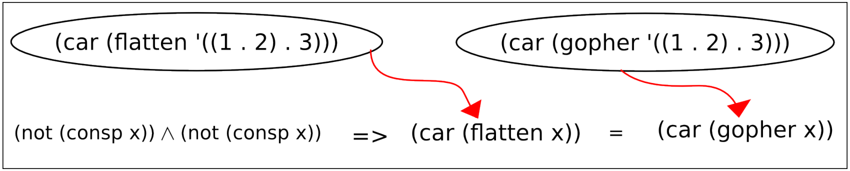
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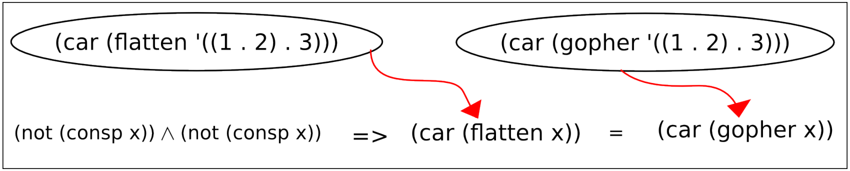
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Our new lemma:

```
(implies (consp x)
         (equal (car (flatten x))
                (car (gopher x))))
```

Current and Future Work

1. New patterns.
2. Heuristics for helping applying induction.
3. Filtering out lemmas.
4. Graphical interface.
5. Empirical evaluation of lemma discovery strategies.