

# Formal Specification and Validation of Minimal Routing Algorithms for the 2D Mesh

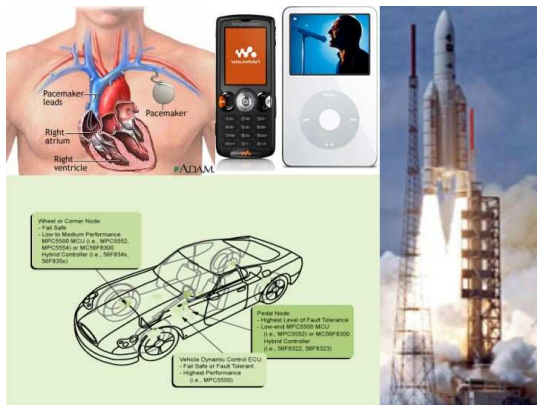
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Part of this work funded by the EU Marie Curie project TAROT

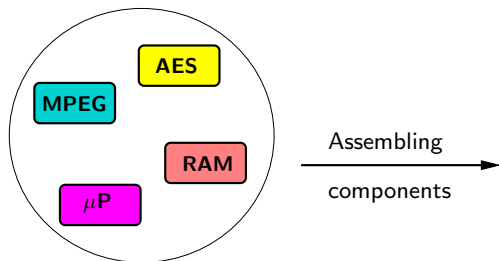
ACL2 2007, Nov. 15-16

# Systems on a Chip



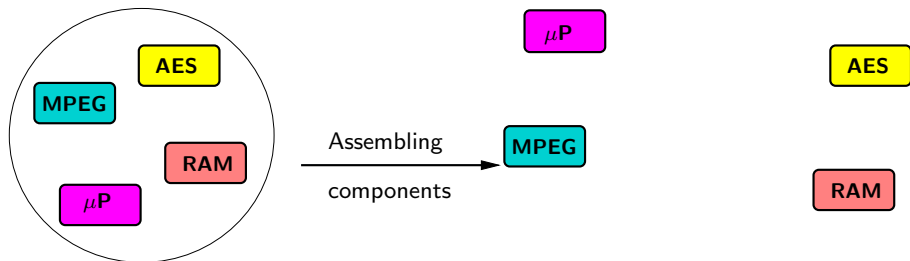
- Everywhere, critical systems
- Ever growing complexity (HW/SW)
- Safety and correctness

# Platform-Based Design and Networks on a Chip



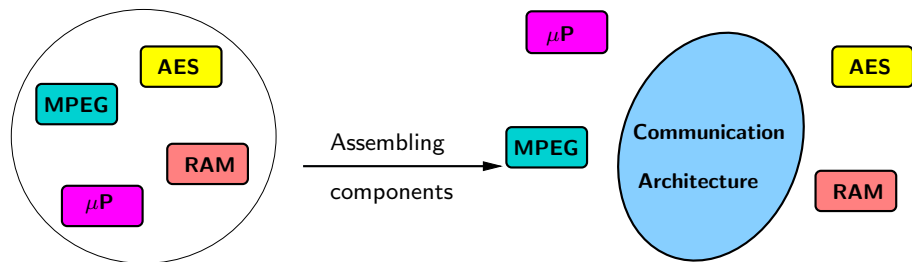
- Re-use of parameterized modules (*Intellectual Properties*)
- High-level of abstraction

# Platform-Based Design and Networks on a Chip



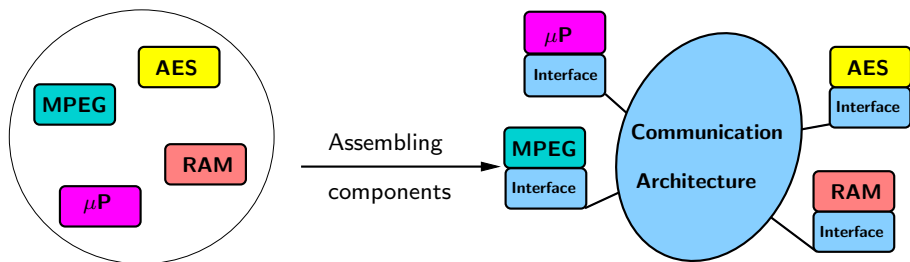
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# Platform-Based Design and Networks on a Chip



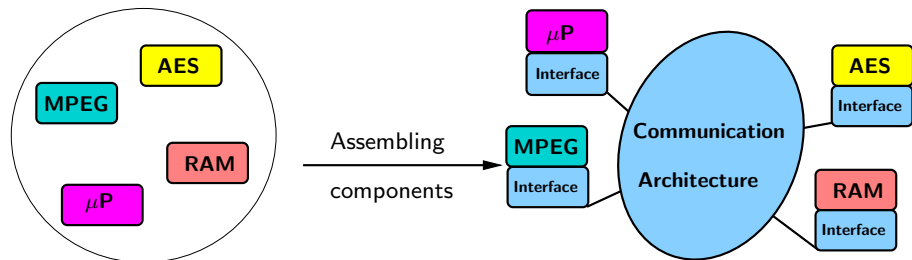
- Re-use of parameterized modules (*Intellectual Properties*)
- High-level of abstraction

# Platform-Based Design and Networks on a Chip



- Re-use of parameterized modules (*Intellectual Properties*)
- High-level of abstraction
- *Communication-centric*: from buses to networks

# Platform-Based Design and Networks on a Chip

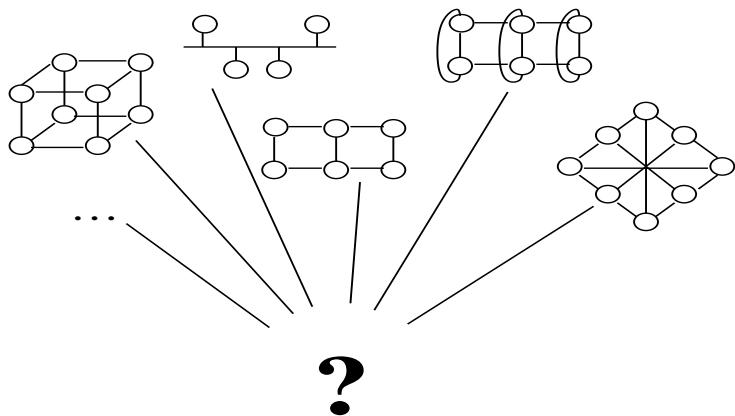


Formal Verification:

- Proof of each component
- Proof of their **interconnection**

# Our Global Objective

Build a *meta-model* of networks: **one** model for **all** architectures



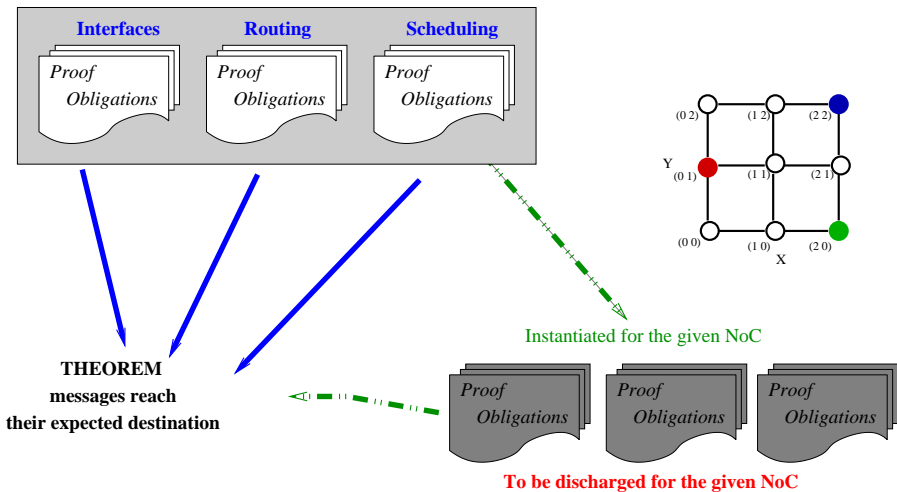


A functional formalism for communications: *GeNoC* (Generic Network on Chip)

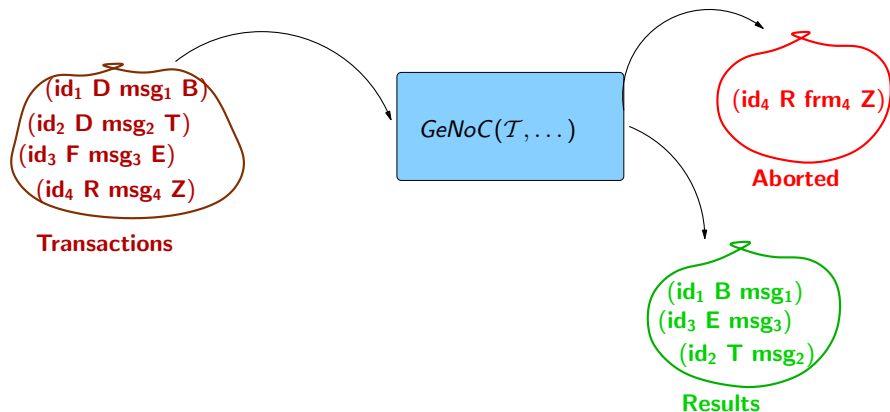
- Identifies the essential constituents and their properties
- Formalizes the interactions between them
- Correctness of the system is a consequence of the essential properties of the constituents
- Mechanized support in ACL2 (see ACL2 2006)
  - Encapsulation allows abstraction
  - Functional instantiation generates proof obligations automatically
- *Minimal routing algorithms for the 2D Mesh*
  - Deterministic case
  - **Adaptive** algorithm

- 1 The GeNoC Model
  - Overview
  - Function GeNoC
  - Proof Obligations
- 2 Routing Function of GeNoC
  - ACL2 Encapsulation
  - ACL2 Constraints
- 3 DoubleY Channel Routing Algorithm
  - Principles and Example
  - ACL2 Function
  - Compliance with GeNoC

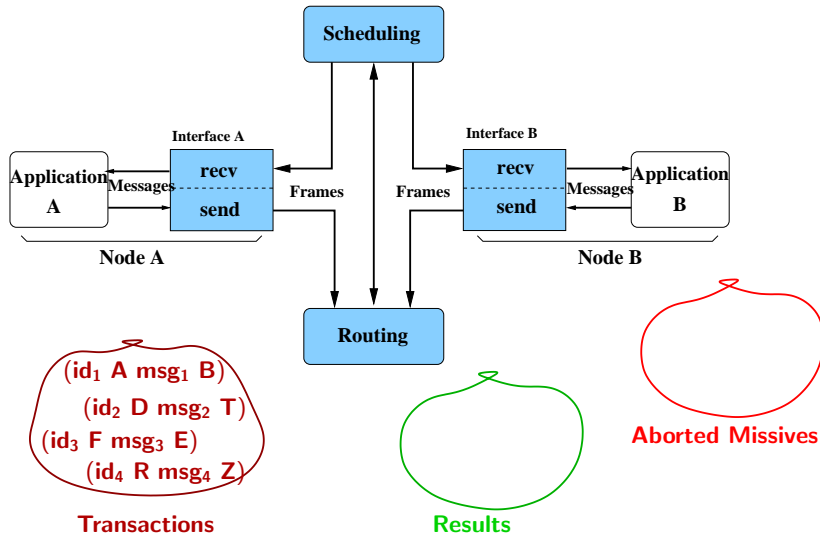
# The GeNoC Approach



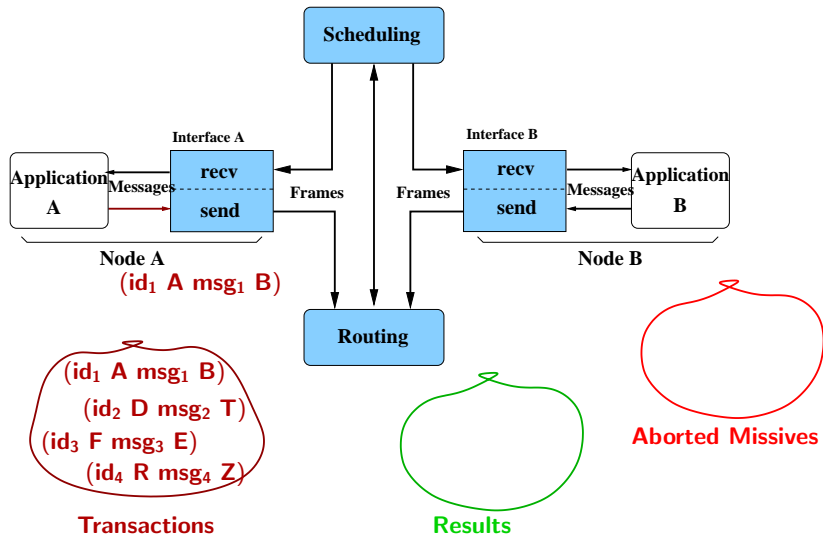
# Overview of Function *GeNoC*



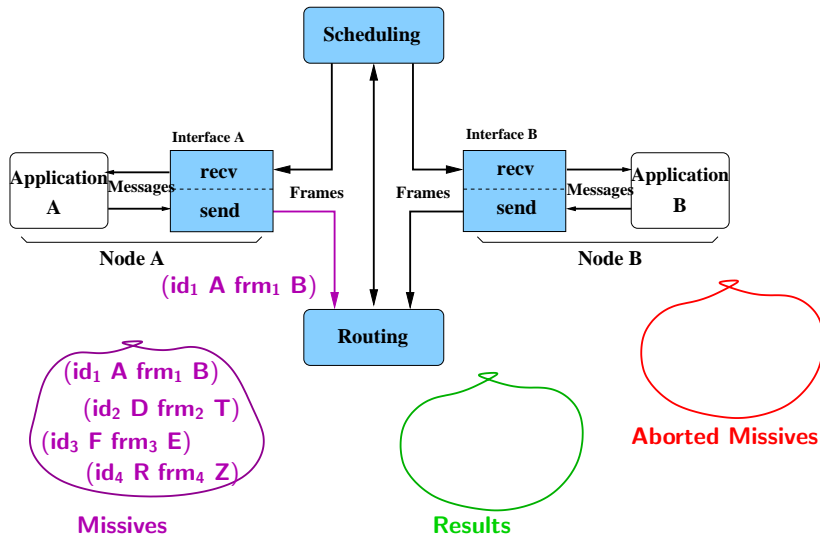
Pending communications are successful or aborted

Function *GeNoC*

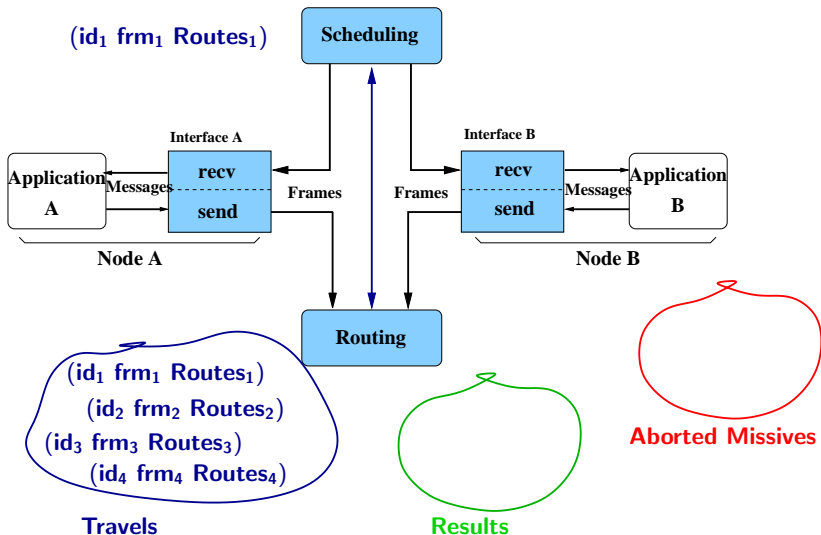
# From Transactions to Missives



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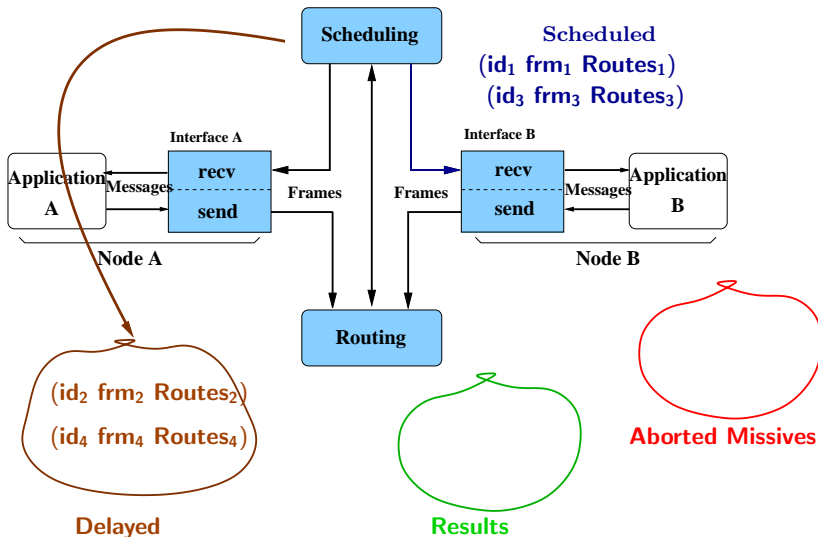


# Routing Algorithm

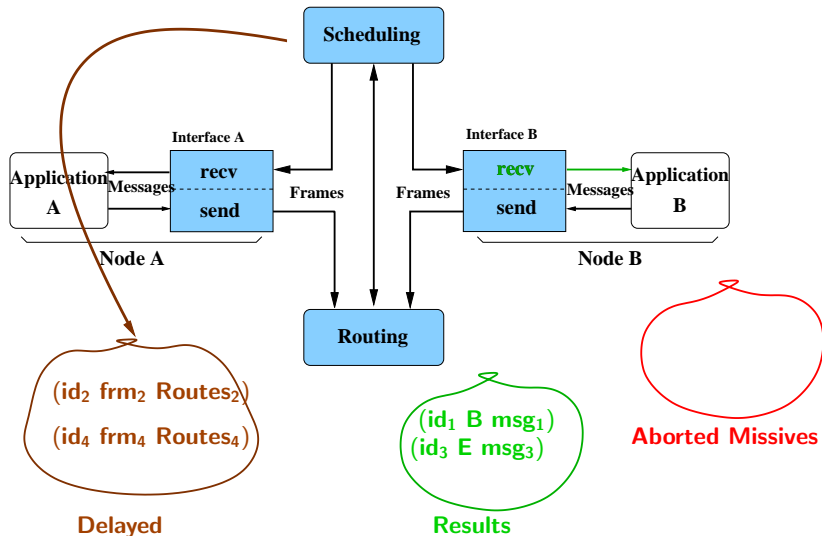




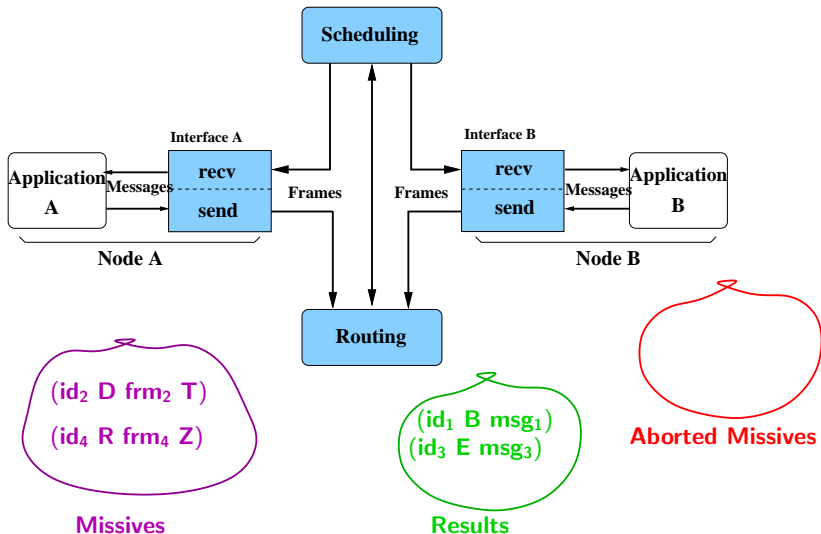
# Scheduling Policy



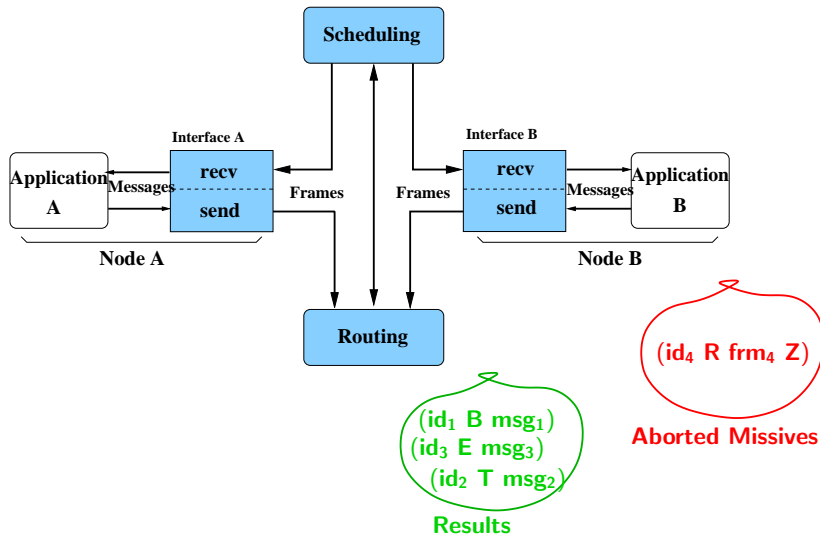
## Results



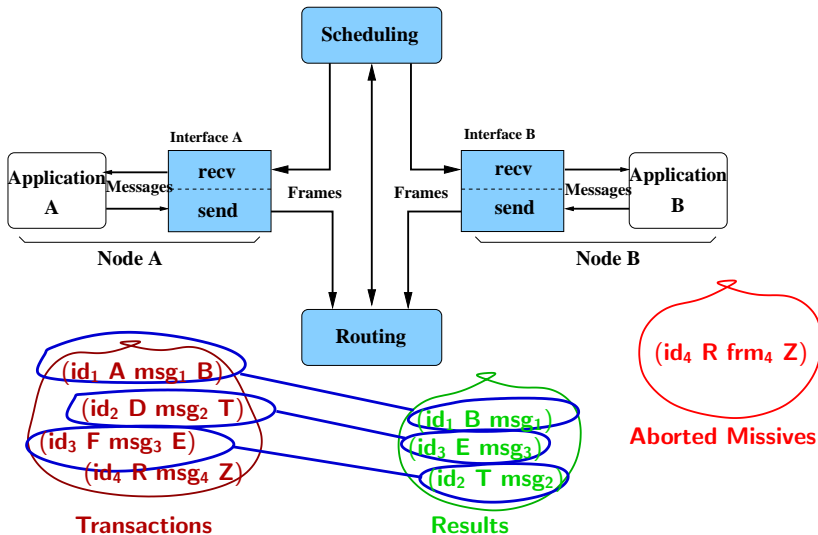
# Aborted Missives



# Aborted Missives



# Correctness Criterion



# Termination

Function *GeNoC* is **recursive**:

- Must be **proved to terminate**
- Associate to every node a ***finite*** number of attempts

# Proof Obligations

- Interfaces
  - The composition  $recv \circ send$  is an identity
- Routing ( $id\ A\ frm\ B$ )  $\mapsto$  ( $id\ frm\ Routes$ )
  - Missive/Travel matching
    - Same frame and identifier
    - Routes effectively go from the correct origin to the correct destination
- Scheduling
  - Mutual exclusion between *Scheduled* and *Delayed*
  - No addition of new identifiers
  - Preserve frames and route correctness

# Proof of the Theorem

- Routing correctness + preserved by scheduling
  - $\rightarrow$  right destination
- No modification on frames
  - $\rightarrow$  every result is obtained by *recv*  $\circ$  *send*
- Interfaces correctness
  - $\rightarrow$  received message = sent message
- Mutual exclusion between *Scheduled* and *Delayed* + no new identifiers
  - $\rightarrow$  cut the proof in two parts



# Outline

- 1 The GeNoC Model
  - Overview
  - Function GeNoC
  - Proof Obligations
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  - ACL2 Encapsulation
  - ACL2 Constraints
- 3 DoubleY Channel Routing Algorithm
  - Principles and Example
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# GeNoC Routing in ACL2

- Generic modules using encapsulation
  - Proof obligations as encapsulated constraints
  - Main constraint expressed by function `CorrectRoutesp`
- Compliance checked via functional-instantiation
  - Instances of proof obligations automatically generated
  - Prove 't' with functional-instantiation hint

# Route Validity

```
( defun ValidRoutep (r m)
  (and (equal (car r) (OrgM m)) ;; 1st = origin
       (equal (car (last r)) (DestM m)) ;; last = destination
       (<= 2 (len r)))) ;; visit at least 2 nodes

( defun CheckRoutes (routes m NodeSet)
  (if (endp routes)
      t
      (let ((r (car routes)))
        (and (ValidRoutep r m)
              (subsetp r NodeSet) ;; use valid nodes only
              (CheckRoutes (cdr routes) m NodeSet))))))
```

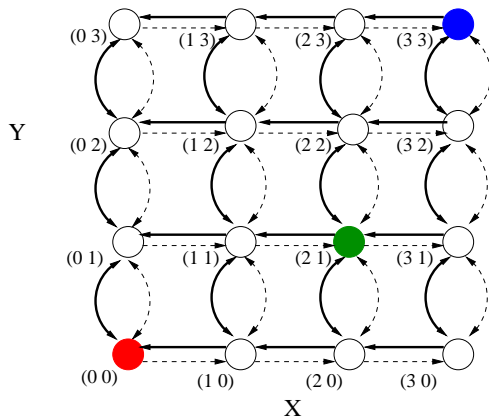
# Route Correctness

```
(defun CorrectRoutesp (V M NodeSet)
  (if (endp V)
      (if (endp M)
          t ;; len(M) = len(V)
          nil)
      (let* ((tr (car V))
             (msv (car M))
             (routes (RoutesV tr)))
          (and (CheckRoutes routes msv NodeSet)
               (equal (IdV tr) (IdM msv)) ;; same id
               (equal (FrmV tr) (FrmM msv)) ;; same frame
               (CorrectRoutesp (cdr V)
                               (cdr M) NodeSet))))))
```

# Outline

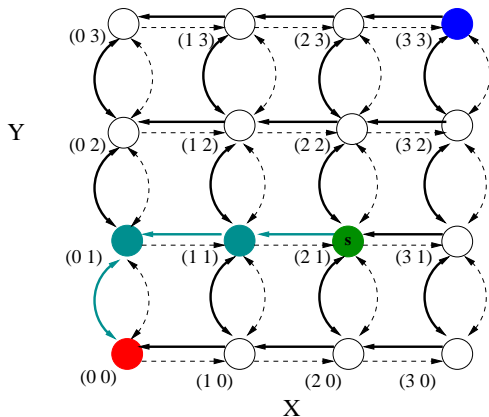
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# DoubleY Channel Routing: Overview



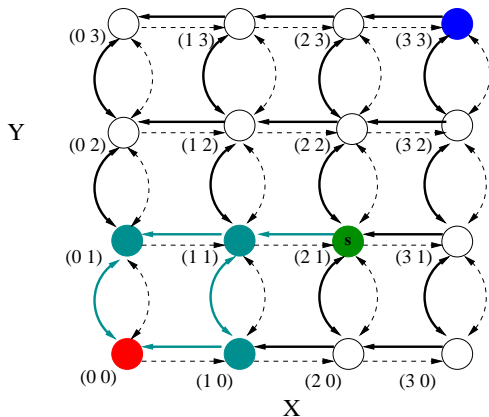
1 x-channel, 2 y-channel  
2 subnetworks

# DoubleY Channel Routing: Overview



1 x-channel, 2 y-channel  
2 subnetworks

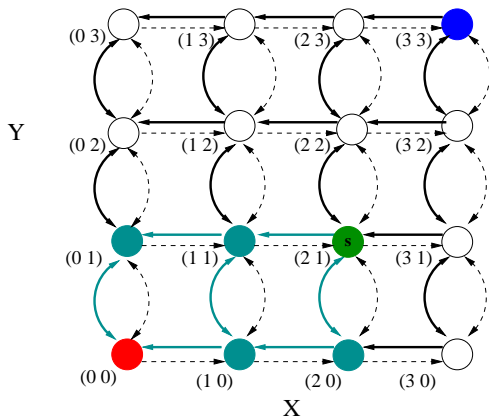
# DoubleY Channel Routing: Overview



1 x-channel, 2 y-channel  
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# DoubleY Channel Routing: Overview

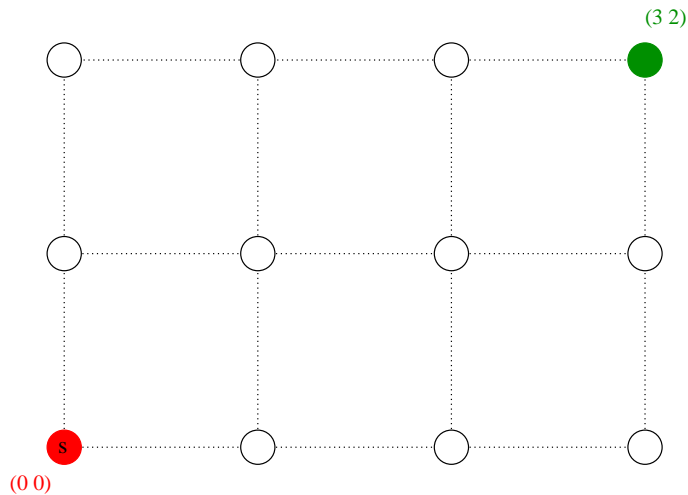


1 x-channel, 2 y-channel  
2 subnetworks

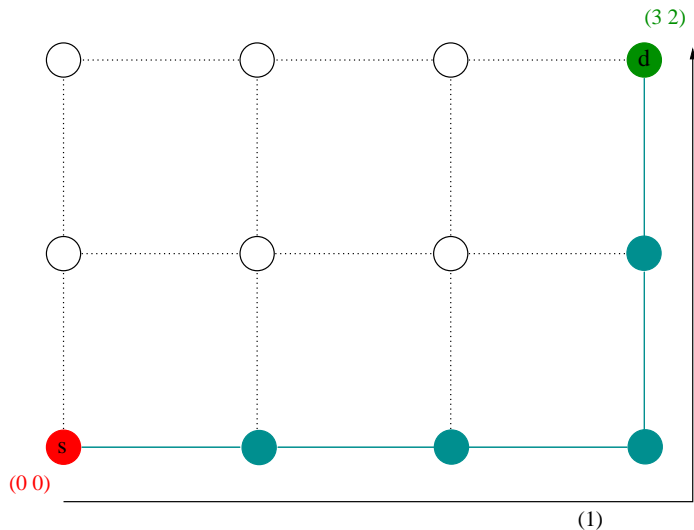
# DoubleY Channel Algorithm: Principles

- Compute **all possible minimal paths** between a source and a destination
- **Alternative** application of XY and YX algorithms
  - Reuse previous proof efforts

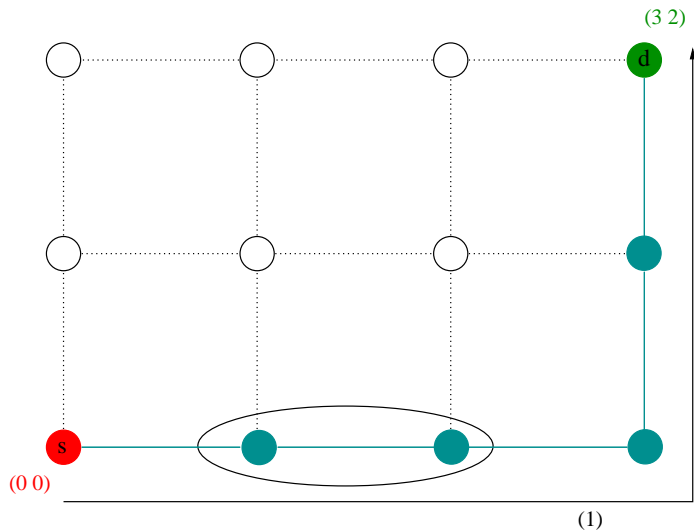
# DoubleY Channel Algorithm: Example



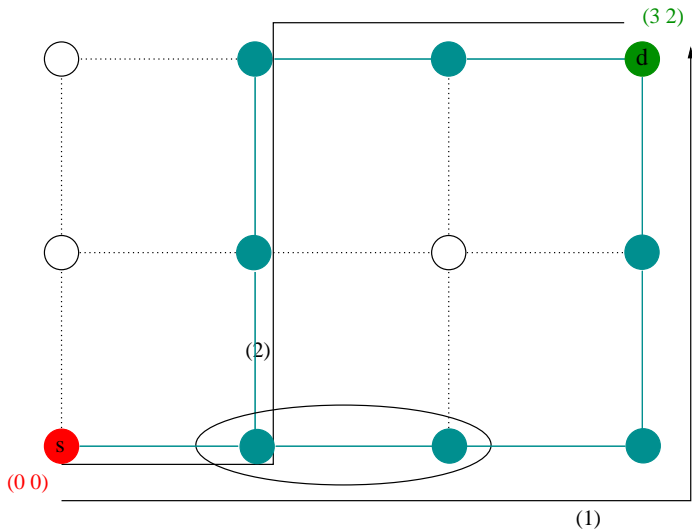
# DoubleY Channel Algorithm: Example



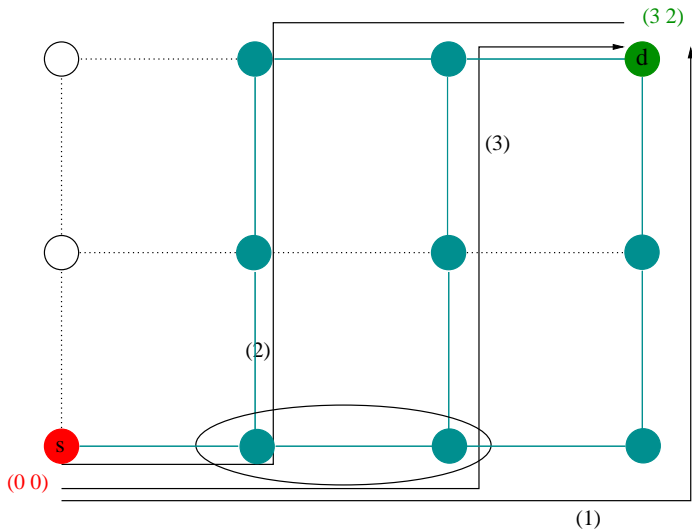
# DoubleY Channel Algorithm: Example



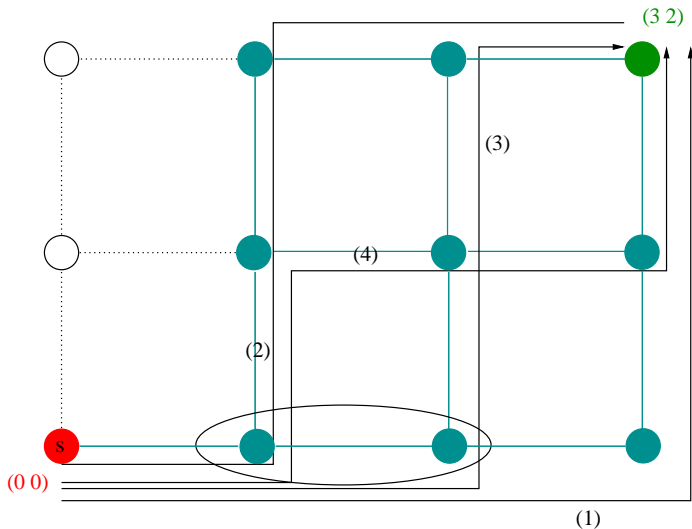
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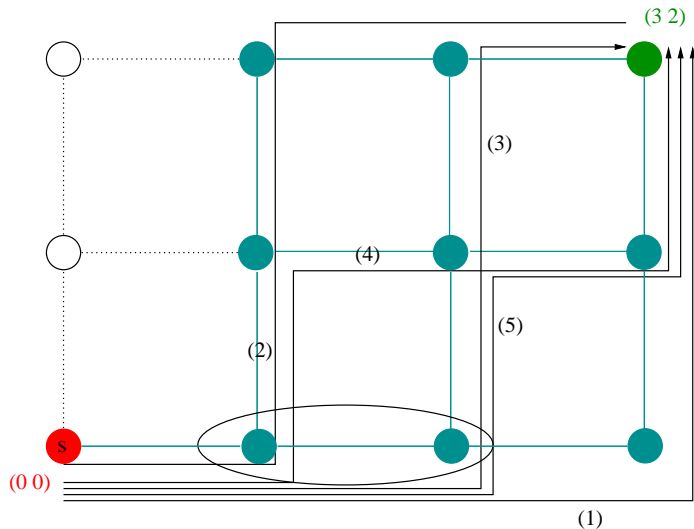


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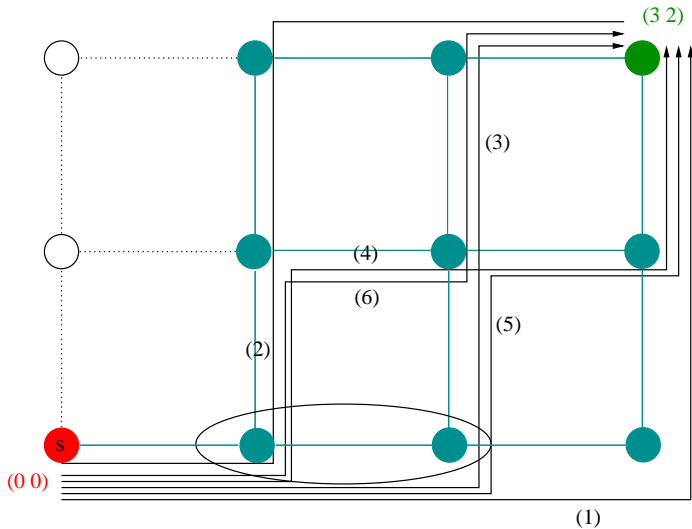




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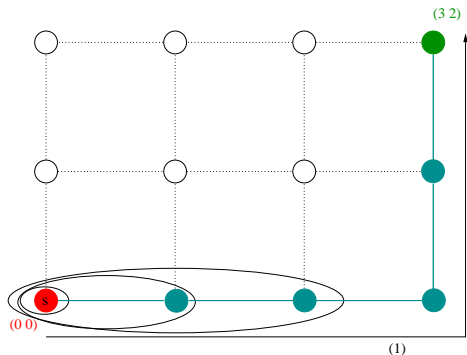


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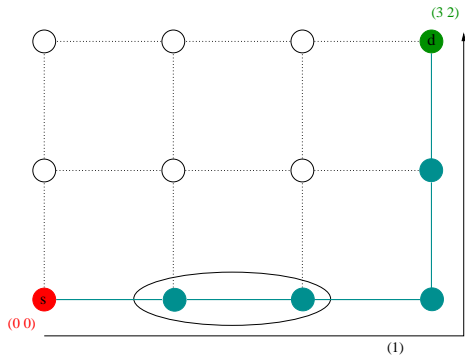
# Prefixes

- Partial routes with nodes without common coordinate with destination
- For a route  $r$  and destination  $d$ , prefixes = (extract-prefixes  $r$   $d$ )



# Sources

- Choice at nodes without common coordinate with destination
- For a route  $r$  and destination  $d$ , sources = (GetSources  $r$   $d$ )



# ACL2 Function `dy1`

```
01 ( defun dy1 (sources d flg prefixes)
02   (declare (xargs :measure
03             (dist (car sources) d)))
04   (if (or (endp sources)
05         (not (CloserListp sources d))
06         (not (2d-mesh-nodesetp sources))
07         (not (coordinatep d))))
08       nil
```

# ACL2 Function `dy1`

```
01 ( defun dy1 (sources d flg prefixes)
    ...
09     (let* ((prefix (car prefixes))
10            (s (car sources))
11            (s_x (car s))
12            (s_y (cadr s))
13            (d_x (car d))
14            (d_y (cadr d))))
```

# ACL2 Function `dy1`

```
01 ( defun dy1 (sources d flg prefixes)
...
15      (cond
16        ((or (equal s_x d_x) (equal s_y d_y))
17          ;; if one coordinate has been reached,
18          ;; we stop
19          nil)
20        (t
21          (let
22            ((routes
23              (cond
```

## ACL2 Function dy1

```
01 ( defun dy1 (sources d flg prefixes)
...
24     (flg
25     ;; last was yx, next is xy
26     (let ((suffix (xy-routing s d)))
29         (cons
30         (append prefix suffix)
31         (dy1 (getSources suffix d)
32             d nil ;; next is YX
33             (append-l-all
34             prefix
35             (extract-prefixes suffix d))))))
36     (t ;; last was xy-routing, next is yx
```



# ACL2 Function `dy1`

```
01 ( defun dy1 (sources d flg prefixes)
    ...
37 -- 43      ;; similar to xy
44          )))))
45      (append routes
46          (dy1 (cdr sources)
47              d flg prefixes)))))))))
```

# Validation of $\text{dy}_1$

Main property: `CorrectRoutesp` for all routes between  $s$  and  $d$

- Routes are subsets of `NodeSet`
- All routes **start with  $s$**  and end with  $d$ 
  - Main invariant: prefixes and sources remember  $s$

```
(defun inv (prefixes sources s)
  (if (endp prefixes)
      (inv-sources sources s)
      (inv-prefixes prefixes s)))
```

# Proof Effort Overview

- 1840 lines of code
  - dy includes definition and validation of yx-routing
  - Almost copy&paste from xy-routing
- Around 70sec. on Intel Dual Core 2400 with 2GB of memory

	defun	defthm	size	time
Mesh NodeSet	8	6	120	0.55
xy-routing	7	44	520	3.8
dy	21	84	1200	67.6

Table: Data for the Double Y Algorithm

# Conclusion

- Application of GeNoC to an **adaptive routing** algorithm
  - **Reuse** of previous work on XY routing
  - Following “The Method”, proof done in few weeks
- Limitations
  - Compute **all** possible paths for **minimal routing** algorithms only
  - Find an algorithm to compute all these paths
- Checking valid instances of encapsulate events
  - No built-in procedure in ACL2
  - **Trick** of proving 't' by functional-instantiation
  - Systematic and **nicer method**  
make-event/defspect.lisp by Ray and Kaufmann (ACL2 v3.2.1)

# Future Work

- **Non-minimal** routing algorithms
- Static **deadlocks**, livelocks, starvation ...
- Dynamic deadlocks: interaction between protocols and interconnect
- **Refinement** method to reach RTL designs
- Explicit notion of **time**
- ...

Learn more during rump session !

# Invariants

First node of prefixes must be the **original source**:

```
(defun inv-prefixes (prefixes s)
  (if (endp prefixes)
      t
      (and (equal (caar prefixes) s)
            (inv-prefixes (cdr prefixes) s))))
```

... first node of sources as well !

```
(defun inv-sources (sources s)
  (if (endp sources)
      t
      (and (equal (car sources) s)
            (inv-sources (cdr sources) s))))
```

# Formal Definition

From a list of **transactions**,  $\mathcal{T}$ , the set of nodes  $NodeSet$  and a list of attempt numbers  $att$ , function  $GeNoC$  produces:

- The list  $\mathcal{R}$  of **results**
- The list  $\mathcal{A}$  for **aborted missives**

$$GeNoC : \mathcal{D}_{\mathcal{T}} \times GenNodeSet \times AttLst \rightarrow \mathcal{D}_{\mathcal{R}} \times \mathcal{D}_{\mathcal{M}}$$

$$(\mathcal{T}, NodeSet, att) \mapsto (\mathcal{R}, \mathcal{A})$$

# Correctness Theorem

$\forall res \in \mathcal{R},$

$$\exists! trans \in \mathcal{T}, \left\{ \begin{array}{l} Id_{\mathcal{R}}(res) = Id_{\mathcal{T}}(trans) \\ \wedge \quad Msg_{\mathcal{R}}(res) = Msg_{\mathcal{T}}(trans) \\ \wedge \quad Dest_{\mathcal{R}}(res) = Dest_{\mathcal{T}}(trans) \end{array} \right.$$

For any result  $res$ , there exists a unique transaction  $trans$  such that  $trans$  and  $res$  have the same identifier, message, and destination.